PROBABILITY

A+

Definition

Probability of event A,

$P(A) = \frac{n(A)}{n(S)}$, $n(A)$ = number of elements in set A

$n(S)$ = number of elements in sample space

= number of total possible outcomes

total number of possible outcome , $n(S) = 12 \times 12 = 144$

Two fair twelve-sided dice with sides marked 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 are thrown, and the numbers on the sides which land face down are noted. Events Q and R are defined as follows.

Q : the product of the two numbers is 24. $Q = \{(2,12), (3,8), (4,6), (6,4), (8,3), (12,2)\}$

R : both of the numbers are greater than 8.

$n(Q) = 6$

(i) Find P(Q). $= \frac{n(Q)}{n(S)} = \frac{6}{144} = \frac{1}{24}$ [2]

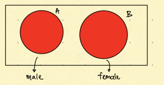
$R = \{(9,9), (9,10), (9,11), (9,12), (10,9), (10,10), (10,11), (10,12), (11,9), (11,10), (11,11), (11,12), (12,9), (12,10), (12,11), (12,12)\}$ $n(R) = 16$

(ii) Find P(R). $= \frac{n(R)}{n(S)} = \frac{16}{144} = \frac{1}{9}$ [2]

(iii) Are events Q and R mutually exclusive? Justify your answer. [2]

(iv) Are events Q and R independent? Justify your answer. [2]

If two events are mutually exclusive, then $P(A \cap B) = 0$



(iii) $P(Q \cap R) = 0 \Rightarrow$ Q and R are exclusive.
excluded

(iv). If two events are independent, then $P(A \cap B) = P(A) \times P(B)$.

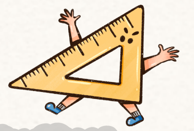
$P(Q \cap R) = 0$ $P(Q) \times P(R) = \frac{1}{24} \times \frac{1}{9} = \frac{1}{216}$

$\because P(Q \cap R) \neq P(Q) \times P(R)$

$\therefore$ events Q and R are not independent.

If two events A and B are mutually exclusive, $P(A \cap B) = 0$

If two events A and B are independent , $P(A \cap B) = P(A)P(B)$



Techniques

Tabulate

Two unbiased tetrahedral dice each have four faces numbered 1, 2, 3 and 4. The two dice are thrown together and the sum of the numbers on the faces on which they land is noted. Find the expected number of occasions on which this sum is 7 or more when the dice are thrown together 200 times. [4]

experiment

2nd die

1st die	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

probability that the sum is 7 or more $= \frac{3}{16}$

$200 \times \frac{3}{16} = 37.5$ occasions.

$n(S) = 16$

Table method : Tabulate / Show all possible outcomes

Permutation & Combination

A bottle of sweets contains 13 red sweets, 13 blue sweets, 13 green sweets and 13 yellow sweets. 7 sweets are selected at random. Find the probability that exactly 3 of them are red. [3]

Permutation & Combination

13 Red → 3

13 Blue, 13 Green, 13 Yellow → 4

$\frac{{}^{13}C_3 \times {}^{39}C_4}{{}^{52}C_7} = 0.176$

total number of ways to choose 7 sweets from 52 sweets

P(red × red × red × ...)

$= \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} \times \frac{39}{49} \times \frac{38}{48} \times \frac{37}{47} \times \frac{36}{46} \times {}^{7}C_3$

$= 0.176$

different ways 3 red sweet chose its position from 7 sweets

35

Conditional Probability

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ or $P(B|A) = \frac{P(B \cap A)}{P(A)}$

* $P(A \cap B) = P(B \cap A)$
* $P(A \cap B) = P(A \text{ and } B)$

When Ted is looking for his pen, the probability that it is in his pencil case is 0.7. If his pen is in his pencil case he always finds it. If his pen is somewhere else, the probability that he finds it is 0.2. Given that Ted finds his pen when he is looking for it, find the probability that it was in his pencil case. [4]

Condition

Conditional probability

- 0.7 pencil case
  - 1 find it
- 0.3 somewhere else
  - 0.2 finds it
  - 0.8 lost it

P(pen is in his pencil case | finds it)

$= \frac{P(\text{pen is in his pencil case and he finds it})}{P(\text{finds it})} = \frac{0.7 \times 1}{0.7 \times 1 + 0.3 \times 0.2} = 0.921$

Multiplicative Probability

Bag A contains 4 balls numbered 2, 4, 5, 8. Bag B contains 5 balls numbered 1, 3, 6, 8, 8. Bag C contains 7 balls numbered 2, 7, 8, 8, 8, 8, 9. One ball is selected at random from each bag.

(i) Find the probability that exactly two of the selected balls have the same number. [5]

Conditional probability

(ii) Given that exactly two of the selected balls have the same number, find the probability that they are both numbered 2. [2]

(iii) Event X is 'exactly two of the selected balls have the same number'. Event Y is 'the ball selected from bag A has number 2'. Showing your working, determine whether events X and Y are independent or not. [2]

(i) P(A & B same) + P(B & C same) + P(A & C same)

$= \frac{1}{4} \times \frac{2}{5} \times \frac{3}{7} + \frac{2}{5} \times \frac{4}{7} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{7} \times 1 + \frac{1}{4} \times \frac{4}{7} \times \frac{3}{5} = \frac{47}{140} = 0.336$

(ii) P(they are both number 2 | exactly two of the selected ball have the same number)

$= \frac{P(\text{they are both number 2 and exactly two of the selected ball have the same number})}{P(\text{exactly two of the selected ball have the same number})}$

$= \frac{\frac{1}{4} \times \frac{1}{7} \times 1}{\frac{47}{140}} = \frac{5}{47} = 0.106$

(iii) X: exactly two of the selected balls have the same number

If two events are independent , then $P(A \cap B) = P(A) \times P(B)$.

Y: the ball selected from bag A has number 2

$P(X \cap Y)$ = P(they are both number 2 and exactly two of the selected ball have the same number)

$= \frac{1}{4} \times \frac{1}{7} \times 1 = \frac{1}{28} = 0.0357$

$P(X) = \frac{47}{140}$ $P(Y) = \frac{1}{4}$ $\Rightarrow P(X) \times P(Y) = \frac{47}{140} \times \frac{1}{4} = 0.0839$

$\because P(X \cap Y) \neq P(X) \times P(Y)$

$\therefore$ events X and Y are not independent.

Tree Diagram

Maria chooses toast for her breakfast with probability 0.85. If she does not choose toast then she has a bread roll. If she chooses toast then the probability that she will have jam on it is 0.8. If she has a bread roll then the probability that she will have jam on it is 0.4.

Condition

(i) Draw a fully labelled tree diagram to show this information. [2]

Conditional probability

(ii) Given that Maria did not have jam for breakfast, find the probability that she had toast. [4]

Tree diagram : Show all possible outcomes

- 0.85 toast
  - 0.8 jam
  - 0.2 not jam
- 0.15 bread roll
  - 0.4 jam
  - 0.6 not jam

Conditional Probability

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(B|A) = \frac{P(B \cap A)}{P(A)}$

$P(A \cap B) = P(B \cap A)$

$P(A \cap B) = P(A \text{ and } B)$

(ii) P(Maria had toast | did not have jam for breakfast)

$= \frac{P(\text{Maria had toast and did not have jam})}{P(\text{Maria did not have jam for breakfast})}$

$= \frac{0.85 \times 0.2}{0.85 \times 0.2 + 0.15 \times 0.6}$

$= \frac{17}{26} = 0.654$